A Dimensionless System for Studying Column Behavior

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ABSTRACT: A dimensionless system is presented for relating certain variables associated with the behavior of columns. This system of plotting is useful when correlating experimental data, studying the effects of such factors as plasticity and shape of cross section, and for checking the designs of columns subjected to eccentric loads.

The conventional system for relating values of average stresses, eccentricity ratios, and slenderness ratios is shown in Figure 1. This system of plotting is of value in certain studies; however, it is limited in its scope. This is so because the behavior of all but very long columns depends upon a number of factors besides slenderness, compressive strength, stiffness, and eccentricity. In some cases, such factors as shape of cross section, loading conditions, shape of stress-strain diagram, and history of loading must be considered when determining failure loads. This is generally the case with many structural columns in the intermediate and short ranges. The dimensionless system of plotting presented in this paper is a powerful tool for evaluating the effects of these factors on column behavior. This system should be of interest to the structural engineer as well as the research engineer.

Attention is directed to the curves shown in Figure 1. Values defined by the Euler curve are associated with elastic buckling of axially loaded columns. Critical stresses beyond the elastic range are defined by means of the tangent-modulus and double-modulus formulas. The Euler formula is valid as long as stresses do not exceed the proportional limit. The tangent-modulus formula defines the load at which bending may begin while the double-modulus formula defines the largest load that an axially loaded column can support. The above formulas are associated with failure conditions of axially loaded columns under idealized conditions. Actually, loading conditions are far from the idealized conditions assumed in theories of column action: bending may begin before the load defined by the tangent-modulus formula is reached. Thus, the maximum axial load for actual columns is usually less than that defined by

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the double-modulus formula and may even be less than that defined by the tangent-modulus formula.

Also shown in Figure 1 is a curve defined by the secant formula and a value of eccentricity ratio equal to 0.50. This curve does not define the buckling load but is associated with conditions which produce maximum stresses in eccentrically loaded columns equal to the proportional limit (or yield point). The average stress at failure may be somewhat larger than that defined by the secant formula. This difference may be great for short columns and is usually negligible for slender columns. This is evident from an inspection of curve *A*, which defines average stresses at failure for steel columns having well-defined yield-points and rectangular cross-sections.

A dimensionless system has been used for plotting the curves shown in Figure 2. Ordinates are ratios of average stresses, $S_a = P/A$, divided by a reference stress S_y. The value of S_y taken for reference is a measure of resistance to yielding or crushing. For steel columns having well-defined yield points, the yield point stress may be used for S_y. For columns which do not have well-defined yield points, the yield strength or proportional limit may be taken for reference. (A convenient value of reference stress is defined later in Figure 3(*b*) by the intersection of a line drawn tangent to the upper portion of the stress-strain curve and the vertical axis of zero strain.)

Abscissas in Figure 2 are ratios of L/r, divided by a reference value of slenderness ratio l/r. The value of slenderness ratio l/r, is defined by the relationship:

$$\frac{l}{r} = \sqrt{\frac{\pi^2 E}{S_a}}$$

The value of l may be considered as the length of a similar column which would fail by Euler buckling at an average stress of $S_a = P/A$. The value of L is the distance between inflection points: In the case of a pin-ended column, L is the actual length. These defined lengths are related as follows:

$$\frac{L}{l} = \frac{L/r}{l/r} = \sqrt{\frac{S_a}{S_E}}\,.$$

This relationship is illustrated in Figure 3.

The curves shown in Figure 1 are re-plotted in Figure 2. It is noted that the Euler relationship defines a straight line in the dimensionless system. Likewise, for constant values of the tangent modulus and double modulus of elasticity, the corresponding relationships define straight lines. In place of the complex curve defined by the secant formula in Figure 1, a simpler curve results when the dimensionless system is used. Examples are discussed below which

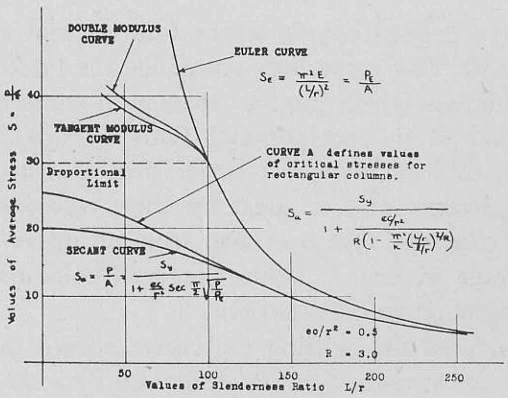

Figure 1

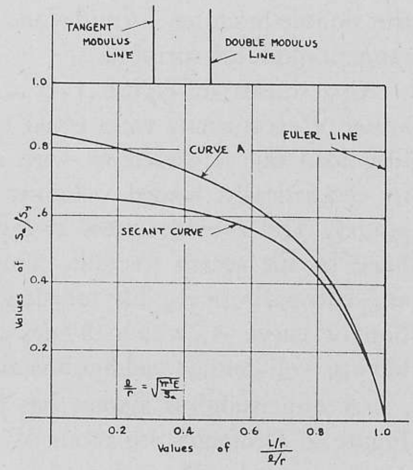

Figure 2

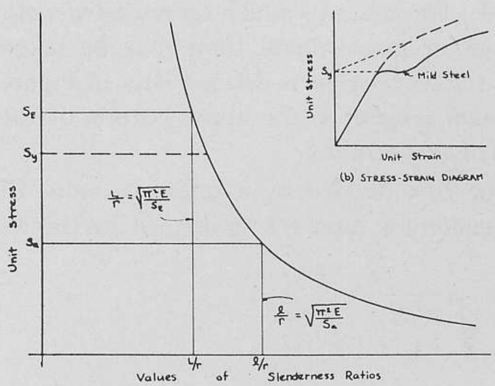

Figure 3

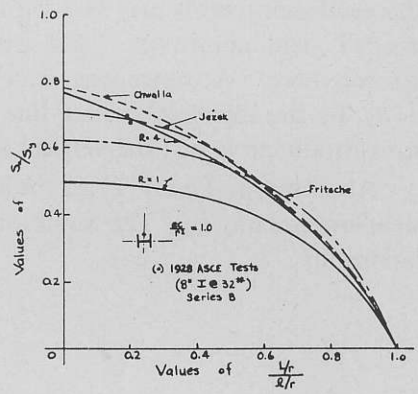

Figure 4

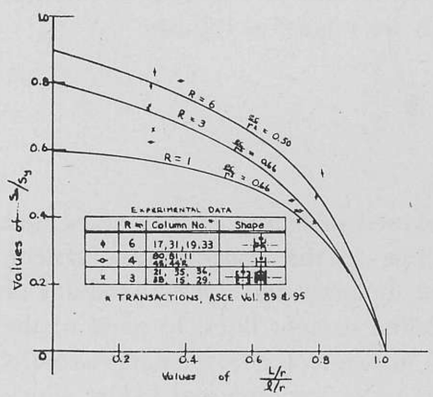

Figure 5

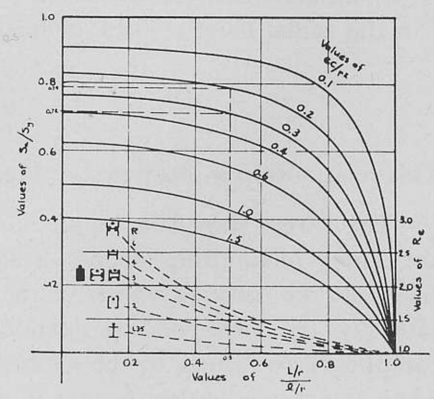

Figure 6

show the merit of using the dimensionless system in studying the behavior of columns.

Example A: Comparison of Results Obtained by Different Investigators of Column Behavior

It is difficult to compare results reported by different investigators when using the conventional system of plotting. This is because each investigator reports results based upon analyses or test results for a particular series of columns. In such cases, shape of cross section, physical properties of the materials, and conditions of loadings usually vary. A critical comparison of such data is possible in the dimensionless system. Shown in Figure 4 are curves which define values of critical stresses for steel columns having I-sections and eccentric loads. The curves shown were obtained principally by means of analytical methods (Jezek, 1937). Also shown are some test data reported by the special committee of the American Society of Civil Engineers on column research (see "Literature Cited"). The difference between the values defined by the curves and the test values is due mainly to differences in the shapes of the stress-strain diagrams near the yield point.

Example B: Studies of Column Behavior and Correlation of Experimental Data

As indicated above, experimental data may be studied to advantage by means of the dimensionless system. Additional data reported by the special committee of the American Society of Civil Engineers on column research are presented in Figure 5 (see "Literature Cited"). The behavior of columns under inelastic conditions is a difficult problem even when idealized conditions are assumed. The "laborious methods of analysis" were considered by the special committee to be "of little practical value for general purposes." In restricting its analyses to the use of the secant formula and elastic conditions, the committee failed to evaluate the effect which shape of cross section has upon critical stresses. In this case, the effect of shape of cross section and the curvature of the stress strain diagram near the yield point can be evaluated from a study of Figure 5. Sketching procedures may be used in many studies where more elaborate procedures are not justified because of the labor required.

Fig. 1.—Comparison of theories of column behavior; conventional system of plotting.
Fig. 2.—Comparison of theories of column behavior; dimensionless system.
Fig. 3.—Reference values of unit stress and slenderness ratios.
Fig. 4.—A comparison of values of critical stresses reported by several writers.
Fig. 5.—A comparison of computed and experimental values of critical stresses.
Fig. 6.—Diagram for checking designs of steel columns.

Example C: Checking the Designs of Steel Columns

The families of curves shown in Figure 6 may be used to check the designs of steel columns (Davis, 1949). The evaluation of the eccentricity, compressive strength, and safety factor in a given problem precedes the use of this diagram. Allowance may be made for these factors when using the diagram as well as for shape of cross section, curvature of the stress strain diagram near the yield point, and the condition of loading.

The family of curves in the upper diagram is defined by means of the secant formula. The inelastic behavior of a column may be expressed or studied in terms of its elastic behavior but modified. This one diagram is adequate for defining critical stresses at initial yielding and at failure for steel columns having different shapes of cross sections. Should a conventional diagram be desired for a particular type of column, such a diagram may be obtained quickly with the aid of the curves shown in Figure 6.

Literature Cited

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