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Polygonalism of an Experimental Crater

Abstract

The shape of a crater produced by a 454-gm charge of TNT buried 152 mm below the surface of a uniform, homogeneous, isotropic medium (sand) was expanded as a Fourier-Bessel series. The coefficients of higher harmonic components show that tendency toward polygonalism need not be a consequence of precrater preferential directionality in the medium.

Introduction

Polygonalism of craters, both lunar and terrestrial, has been known for a long time. A fairly extensive list of references can be found, e.g., in Fielder (1961, p. 180; 1965, p. 59).

Speculations as to the causes of polygonalism either advocate precrater preferential directions, or crustal stresses and alterations following crater formation (Fielder, 1961, p. 180; 1965, p. 140, 149) or a combination of the two.

While the above interpretations may be correct, we want to show that neither need be a necessary condition for polygonalism. Our analysis was carried out on an experimental crater produced by a detonation of a spherical charge of 454 gm of TNT in a homogeneous, isotropic medium of virtually zero cohesion. The specifications of the crater were taken from C. V. Fulmer (1965) and are reproduced in Table 1. The crater was surveyed immediately following detonation so that no postcrater changes were introduced.

Our results show that this crater shows a significant tendency toward "squareness," and a slight one toward "pentagonalism."

Analysis

The shape of the present crater is shown as a contoured map in Figure 1. This shape was converted into a digital representation in polar coordinates r, ϑ along 120 radial profiles, 3° apart. Along each profile the shape was evaluated at 40 mm intervals to a distance of 3.92 m. The origin of the polar coordinate system was directly above the center of the charge. Accuracy in determination of the vertical relief is of the order of $\pm$ 1 mm (Fulmer, 1965).

The digitalized crater shape, $\xi\ (r, \vartheta)$, was then expanded as a Fourier-Bessel series, (Bronwell, 1953, p. 82; Gray and Mathews, 1966, p. 111)

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TABLE 1. Crater Specifications-D4a.

Explosive	A. Sphere of TNT, 454 gm B. Depth of burial, 152 mm C. Detonation energy, 2.09 x 10^{13} ergs
Medium	A. Well-packed, fine-grained, kiln-dried plaster sand 1. Moisture percent, 0.5 2. Density, 1.6 gm/cm^3 B. Sand grain size: 100 percent smaller than #8 screen, 98 percent smaller than #16 screen, 66 percent smaller than #30 screen, 11 percent smaller than #50 screen, and 3 percent smaller than #100 screen.
Test Site	A. Test-pit sand medium was compacted with the aid of a Jackson Mechanical Compactor capable of delivering blows of 15 psi, to a density of approximately 1.7 gm/cm^3. B. Entire pad area was leveled to a plane surface of ± 2mm. C. Area 5 of The Boeing Co. Tulalip test site, Marysville, Washington. D. Area was 7.62 m square and 1.83 m deep, underlain by a gravel-filled sump.
Crater	A. Maximum crater depth = 191 mm B. Average crater radius = 648 mm C. Maximum crater radius = 682 mm D. Average X-Y height = 39 mm E. Average X-Y lip-to-lip diameter = 1850 mm F. Crater volume = 117 x $10^6$$mm^3$
Topographic Measurements	Topographic measurements were taken with a graduated track, bridge, and vertical probe assembly. The crater survey recorded the postshot relief by measuring along traverses parallel and normal to the tracks. Postshot elevation values were recorded and used in the construction of contour maps.

$$\xi(r, \vartheta) = \tfrac{1}{2} \sum_{s=1}^{S} A_{os} J_o(k_{os} r) + \sum_{n=1}^{N} \sum_{s=1}^{S} (A_{ns} \cos n\vartheta + B_{ns} \sin n\vartheta) J_n(k_{ns} r) \qquad (1)$$

where

$$\begin{pmatrix} A_{ns} \\ B_{ns} \end{pmatrix} = \frac{2}{\pi a^2 [J_{n+1}(k_{ns} a)]^2} \int_o^{2\pi} d\vartheta \int_o^{a} f(r, \vartheta) \begin{pmatrix} \sin n\vartheta \\ \cos n\vartheta \end{pmatrix} J_n(k_{ns} r)\, r dr \qquad (2)$$

Here J_n is the n-th order Bessel function, and a is the radial distance at which the relief is assumed to die out. N and S should be infinite, but for practical reasons, the summation was truncated at

$$N = S = 12 \qquad (3)$$

The function $f(r,\vartheta)$ is the "true" crater shape, whereas $\xi(r,\vartheta)$ is its digital representation. The coefficients k_{ns} were determined so that the n-th order Bessel function has its s-th root at $r = a$.

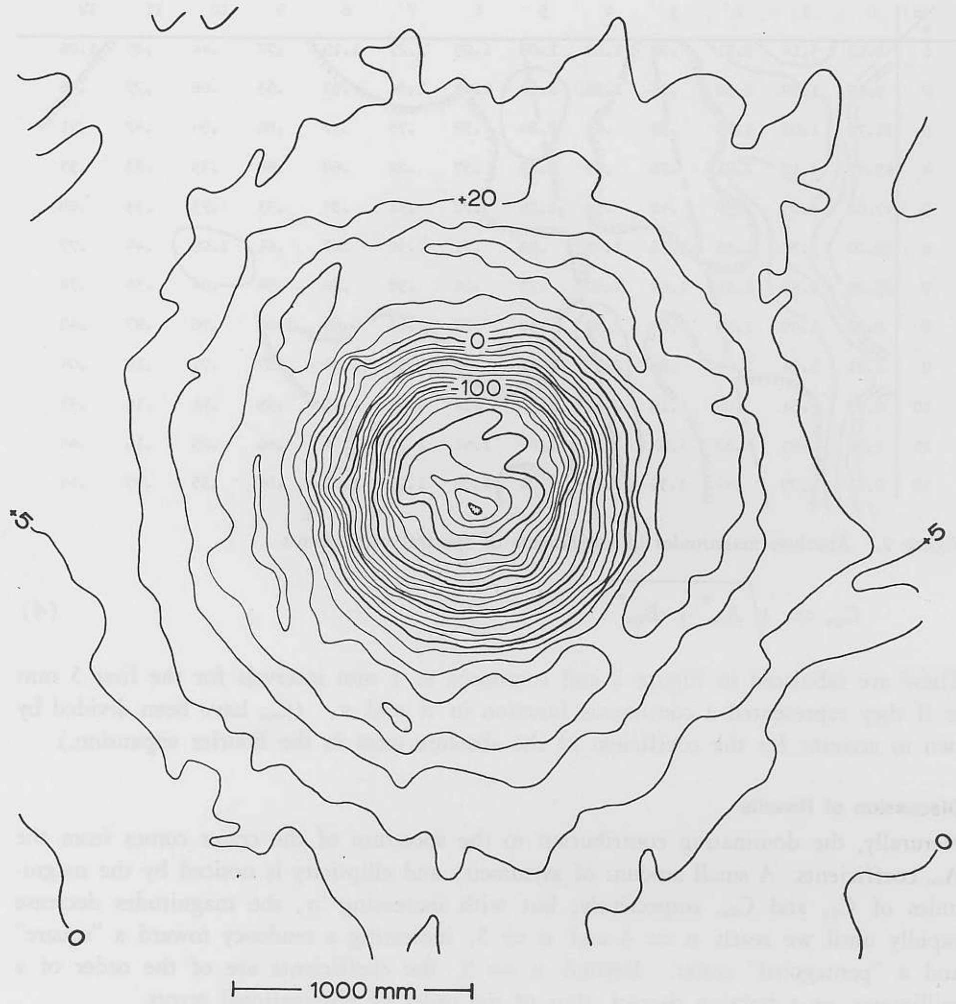

Figure 1. Contoured map of crater. (Elevations in millimeters. Contour interval 10 mm, except the discontinuous 5 mm at the periphery of the figure.)

If the crater were circularly symmetrical, only the A_{0s} would be finite; the rest of the A_{ns} and B_{ns} would vanish. If, on the other hand, the crater deviates from circular symmetry, those coefficients are also finite. An "elliptic" crater will have a significant contribution to its spectrum in the A_{2s}, B_{2s}; a "square" one in A_{4s}, B_{4s}; a "hexagonal" one in A_{6s}, B_{6s}; and so forth. Likewise, if a crater is fairly circular in the central portion, but begins to show polygonalism farther away from the center, the contribution will be noticeable in the lower values of s; otherwise, in the higher ones.

Equation (2) was solved numerically on the General Electric 635 digital computer.

Since it is the absolute value of the coefficients that gives the information about polygonalism, we derived the coefficients

s \ n	0	1	2	3	4	5	6	7	8	9	10	11	12
1	2.69	3.14	2.51	.78	1.68	1.69	1.09	1.27	1.19	.72	.44	.85	1.08
2	9.65	1.59	2.38	.25	1.26	2.35	1.03	1.34	.93	.63	.66	.77	.96
3	31.70	1.02	1.05	.52	.65	2.24	.38	.73	.37	.86	.94	.47	.71
4	45.67	1.10	2.01	.78	.99	1.99	.37	.24	.60	.69	.35	.23	.55
5	47.02	3.05	.57	.42	.58	1.25	.72	.31	.37	.33	.73	.33	.68
6	38.20	3.98	.38	.75	1.05	.68	.71	.36	.47	.61	1.09	.49	.77
7	23.25	2.69	1.14	1.62	1.90	.73	.26	.58	.26	.94	.94	.38	.74
8	8.22	1.49	1.49	.78	2.30	.50	.20	1.23	.69	1.00	.76	.07	.49
9	2.21	1.24	1.04	.68	2.57	.61	.54	1.58	1.04	.87	.71	.16	.04
10	6.93	.61	.02	1.19	2.87	1.30	1.12	1.52	1.01	.59	.51	.16	.33
11	7.50	.45	.60	1.62	2.81	1.07	1.41	1.29	.94	.46	.25	.16	.44
12	6.11	.79	.49	1.35	2.46	.70	1.08	1.25	1.02	.84	.35	.05	.44

Figure 2. Absolute magnitudes of coefficients of spectral components.

$$C_{ns} = \sqrt{A_{ns}^{2} + B_{ns}^{2}} \qquad (4)$$

These are tabulated in Figure 2 and contoured at 1 mm intervals for the first 5 mm as if they represented a continuous function in n and s. (C_{os} have been divided by two to account for the coefficient of the absolute term in the Fourier expansion.)

Discussion of Results

Naturally, the dominating contribution to the spectrum of the crater comes from the A_{os} coefficients. A small amount of asymmetry and ellipticity is noticed by the magnitudes of C_{1s} and C_{2s}, respectively; but with increasing n, the magnitudes decrease rapidly until we reach $n = 4$ and $n = 5$, indicating a tendency toward a "square" and a "pentagonal" crater. Beyond $n = 5$, the coefficients are of the order of a millimeter, or a fraction thereof, thus of the order of observational errors.

It may be of some significance that both the low and high trend on Figure 2 (heavy lines) are slightly inclined toward the east of north. It may mean that, closer to the center, the tendency is toward a lower order of polygonalism, whereas farther away, the shape tends to break up toward a higher number of lobes.

Figure 3 represents the square roots of sums of squares of the columns of Figure 2:

$$D_n = \sqrt{\sum_{s=1}^{12} (A_{ns}^{2} + B_{ns}^{2})} \qquad (5)$$

(Here, the D_o has been omitted because of its much greater magnitude.)

Since elevation data are reliable to approximately ± 1 mm, the level of significance of those sums is about 3.46 mm. The results are obvious: the eccentricity of the crater and the "squareness" are of approximately the same magnitude; the ellipticity and pentagonalism, while lower than the above contributions, are significant; the rest of the harmonic components are on the verge of, or below, the level of significance.

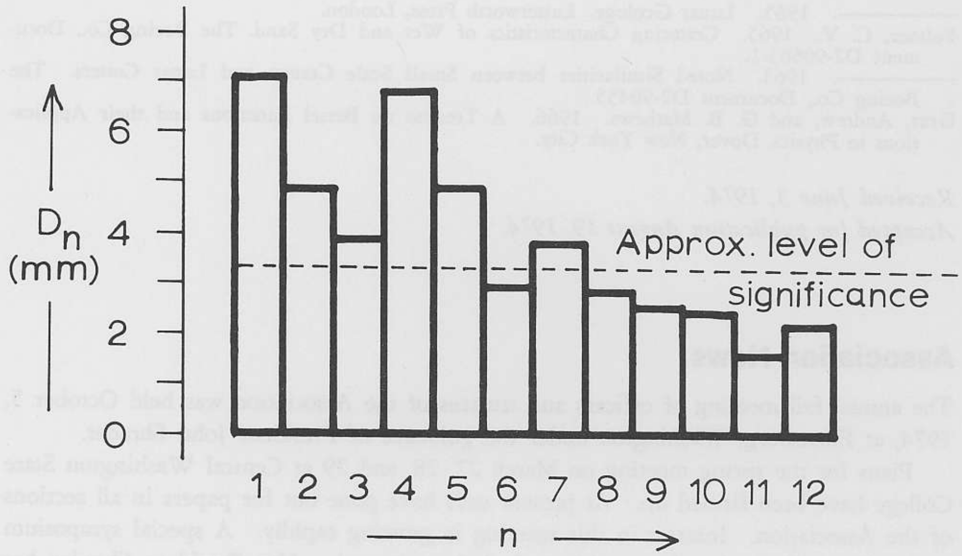

Figure 3. Square roots of sums of squares of coefficients of harmonic components versus n :

$$D_n = \sqrt{\sum_{s=1}^{12} (A_{ns}^2 + B_{ns}^2)}$$

Conclusion

Since there does not seem to be any reasonable possibility that the medium in which the above crater was generated had any precrater preferential directionality, and since the coefficients of some of the higher harmonics are well above the level of significance, we conclude that precrater directionality is not a necessary condition for crater polygonalism.

That, of course, opens the question as to what mechanism is responsible for the presence of higher harmonics in the crater shape. While an answer to the above question is not a part of the present problem, we speculate that the interface between the ambient medium and the expanding gasses created by the detonation may become Taylor unstable during the later stages of the expansion process and that crater "polygonalism" may be a result of stress concentration and jetting along preferential azimuths of the fast-growing higher harmonics perturbation.

Acknowledgements

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Association News

The annual fall meeting of officers and trustees of the Association was held October 5, 1974, at Ellensburg, Washington under the guidance of President John Shrader.

Plans for the spring meeting on March 27, 28, and 29 at Central Washington State College have been firmed up. At present calls have gone out for papers in all sections of the Association. Interest in this meeting is growing rapidly. A special symposium concerning refinery location problems is being planned. Also President Shrader has arranged for Mr. Don Porter to entertain following the banquet.

Other business of special interest to members is the proposed increase in dues—to $7.00 for regular members and $3.00 for students, with a $1.00 surcharge if paid after January 1, 1976. This will be decided upon by the members next spring.

A membership drive is under way and each member is being asked to recruit one new member this year.

Please be reminded to send your 1975 dues to R. Naskali, Department of Biological Sciences, University of Idaho, Moscow, Idaho 83843—$5.00 for regular members and $2.00 for students.

Milton Mosher
Secretary