John E. Mitchell[1]
and
Charles R. Hatch
College of Forestry, Wildlife, and Range Sciences
University of Idaho
Moscow, Idaho 83843

Use of Polar Coordinates in Random Sampling[2]

Abstract

A method is presented that allows field workers to locate random points quickly through the conversion of Cartesian coordinates to polar coordinates using a pocket calculator.

Introduction

In sampling natural populations for a given attribute (e.g., density), randomness must be introduced into the sample design if estimates of error are to be assessed and valid conclusions obtained. This concept of randomness is now generally accepted by ecologists. There are numerous examples every year in ecological research, however, in which samples are collected using either a convenience or a judgment selection process (Lapin, 1975) in spite of the limitations inherent in such a strategy. Paramount among the justifications for such sampling is the apparent belief that, in some circumstances, estimates of the mean will deviate less from the true value than comparable estimates given by random samples (Greig-Smith, 1964), and a conviction that random sampling is too complex and time consuming (Cochran, 1963).

In this paper, we present a simple method that allows a field worker to locate sampling points using a systematic technique; i.e., by moving a given distance (r) in a given direction (α). This technique diminishes the basis for the argument against random sampling that it is inefficient. The other criticism, less deviation from the mean, has been shown to be only marginally valid at best (Bourdeau, 1953).

When a population is being sampled randomly, every possible sample point in the population has an equal probability of being selected. Accomplishment of this goal is not as elementary as is sometimes believed. For example, the customary manner of throwing a quadrat in some arbitrary direction as a way of achieving random observations is ineffectual (Greig-Smith, 1964). A systematic random sample, in which the first sample point is selected randomly and the remainder are chosen at even-spaced intervals, is also prone to sampling bias, especially if the interval coincides with natural periodic variation (Cochran, 1963).

One way to obtain random samples has been through the use of Cartesian coordinates.

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[1]Present address: USDA Forest Service, Rocky Mtn. Forest and Range Exp. Stn., Fort Collins, Colorado 80524.

[2]Published as Forest, Wildlife, and Range Experiment Station Contribution No. 90, University of Idaho, Moscow 83843.

This function requires that orthogonal axes be constructed in a manner that all possible sample points in the population are contained in the plane defined by the two axes. To locate a sample point, a value is randomly selected for each axis. The sample point is then located by going along one axis to the value chosen for it and then traversing at a right angle a distance equal to the value selected for the second axis. It is generally necessary to return to the first axis or the origin of the two axes before locating the next sample point. Thus, substantial time and effort are wasted between location of sample points.

Conversion to Polar Coordinates

An alternative way of defining the location of points on a plane is through the use of polar coordinates. Using this system, the location of a point is defined in terms of a distance from the origin (r) and a directed angle from an initial ray emitting from that origin (Θ). The following equations and Figure 1 illustrate the relationship between Cartesian and polar coordinates.

$$x = r \cos \Theta$$

$$y = r \sin \Theta$$

Thus,

$$r = \sqrt{x^2 + y^2} \quad (1)$$

$$\Theta = \tan^{-1} y/x \quad (2)$$

where: x and y are Cartesian coordinates
r is the distance from the origin (0,0) to the point defined by Cartesian coordinates (x,y)
Θ is an angle between 0 and 90 degrees.

Although a sample point location is defined with respect to a Cartesian coordinate system, polar coordinates could be used to define the location of the next sample point in terms of the location of the current sample point. Thus, an individual need not return to an axis or the origin of both axes before locating the next sample point, which could result in a substantial savings of time and effort.

The location of the next sample point in terms of the location of the current sample point can be accomplished in the following manner and is illustrated in Figure 2. The horizontal and vertical distance between the location of the current and the next sample point can be computed as follows:

$$\Delta x = x_{j+1} - x_j \quad (3)$$

$$\Delta y = y_{j+1} - y_j \quad (4)$$

where: (x_{j+1}, y_{j+1}) is the Cartesian coordinate associated with the next sample point location.
(x_j, y_j) is the Cartesian coordinate associated with the current sample point location.

Equation 1 can be used to express Δx and Δy in terms of a polar coordinate distance (r). Equation 2 and Table 1 can be used to define a bearing (∝) from the current sample point location to the next sample point location. This table assumes magnetic north is used as the initial ray. Therefore, the angle being measured is with respect to the vertical axis of the Cartesian coordinate system (Fig. 2).

TABLE 1. Computation of the bearing from the current sample point location to the next sample point location.

If the sign of		The bearing
x computed using equation 3 is	y computed using equation 4 is	α[1] is
positive	positive	$\|\Theta\|$[2]
negative	positive	$180 - \|\Theta\|$
negative	negative	$180 - \|\Theta\|$
positive	negative	$360 - \|\Theta\|$

[1]Azimuth in degrees.
[2]Absolute value of Θ computed using Equation 2.

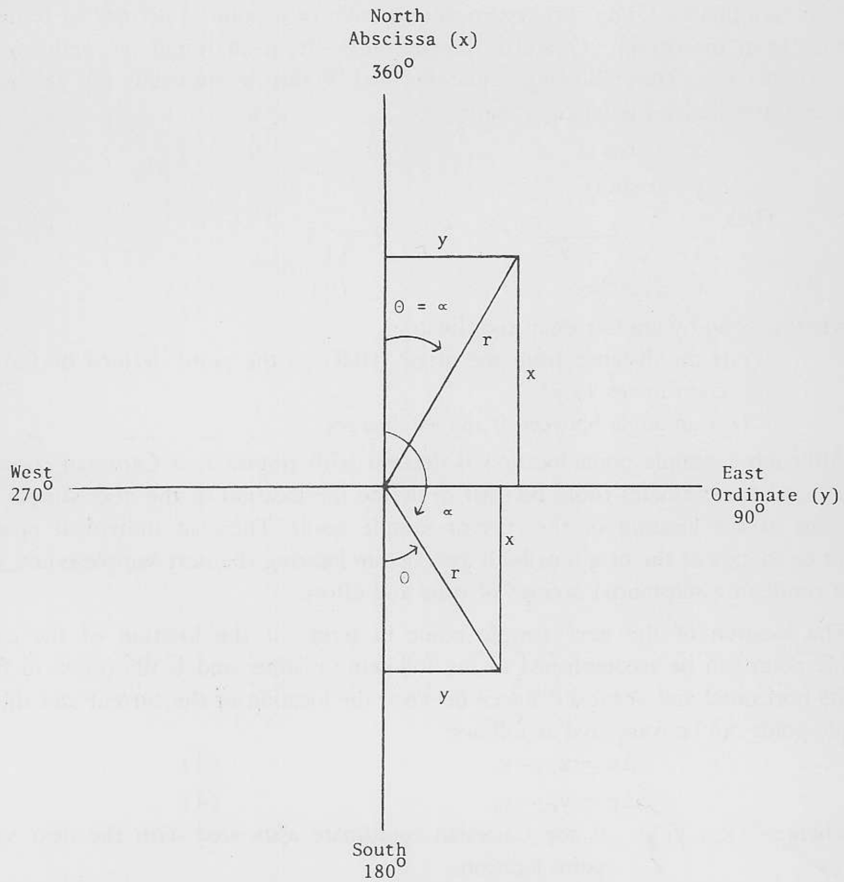

Figure 1. Example of relationship between polar and Cartesian coordinate system for two points in the 1st and 2nd quadrats, respectively.

Sample Point Location Generation

If every possible sample point in the population is to have an equal probability of selection, values along the X and Y axes must all have the same chance of being selected. This can be accomplished through the use of a random number table. A pocket

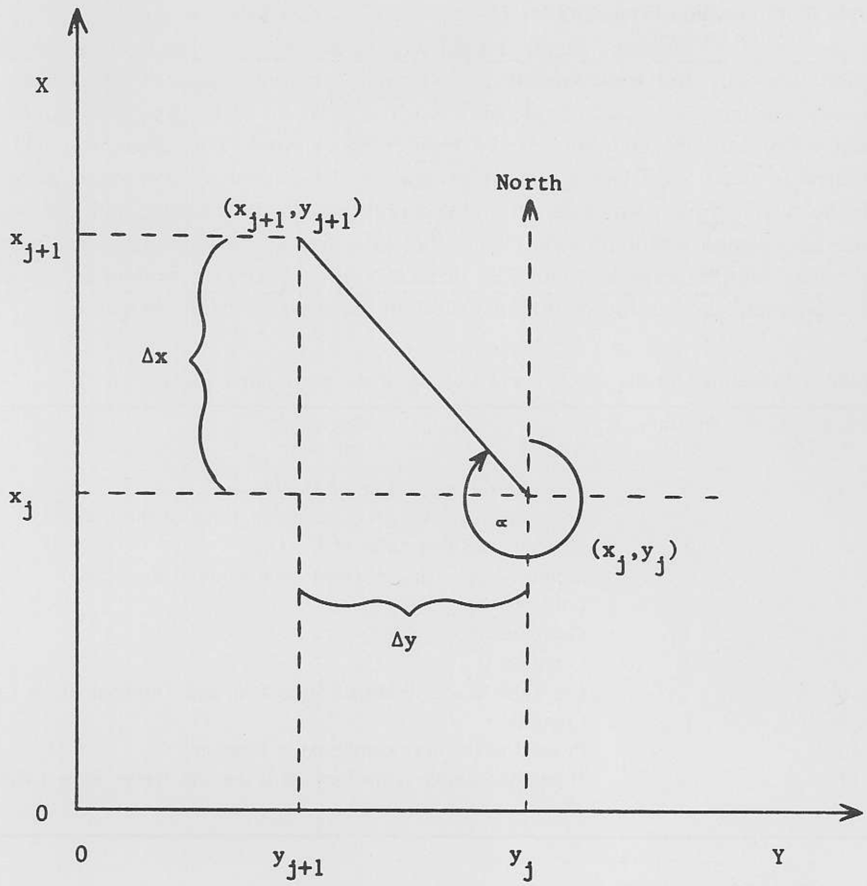

Figure 2. Geometric depiction of use of polar coordinates to locate the next sample point with respect to the present sample point.

calculator is also capable of generating uniformly distributed pseudorandom numbers.[3] The numbers lie in the interval 0 to 1 and are generated through the use of the following recurrence relationship (Martin, 1968):

$U_{i+1} = KU_i$

where: $K = 8t \pm 3$

t is a positive nonzero integer

U_i is the last decimal number generated between 0 and 1. When initialized it should be an odd value.

U_{i+1} is the least significant part of $K \bullet U_i$ and is the next decimal number generated between 0 and 1.

By multiplying U times the length of a Cartesian axis (X), a value along the X axis is randomly generated from a uniform distribution.

[3]Hewlett-Packard HP-45 Application Book, Hewlett-Packard Co., 1974, p. 165.

Sample Point Location Procedure

The preceding relationships enable a field worker to generate his next sample point location randomly and walk directly to it from his current location. To perform the necessary calculations, a pocket calculator with at least an arctangent and square root function key and one constant storage register is required. The procedure, which is presented in detail in Table 2, entails generating the Cartesian coordinates associated with the next sample point location and computing both the distance and the bearing to the next sample point location. With this information, the individual can proceed to the next sample point location. The distance can be paced or, preferably, measured with a tape and the bearing maintained through the use of a hand compass.

TABLE 2. Procedures for the selection and location of the next sample point.

Step No.	Equation No.	Description of Step
1	5	Generate the next value of U (U_{i+1})
2		Compute $x_{j+1} = U_{i+1}$ • (maximum value of the x axis)
3	5	Generate the next value of U (U_{i+2})
4		compute $y_{j+1} = U_{i+2}$ • (maximum value of the y axis)
5	3	Compute Δx
6	4	Compute Δy
7	2	Compute Θ
8		Use Table 1 and results of Steps 5, 6, and 7 to compute $\propto$
9	1	Compute r
10		Proceed to the next sample point location
11		If another sample point location is desired, repeat Steps 1 through 10.

Discussion and Conclusions

The technique described above is not without limitations. Rugged terrain and/or heavy overstory vegetation could preclude locating predetermined sampling points by a sighting and pacing system. However, such conditions would also preclude the ground location of predetermined sampling points by alternative point location procedures.

The magnitude of any biases which might be inherent in using a polar coordinate system in comparison with a Cartesian coordinate system has not yet been assessed. One would expect to find no difference since similar measure problems exist with both methods. Furthermore, regardless of the system used, measurement to sample point locations need not be exact to retain the principle of randomness (Greig-Smith, 1964).

It is possible to generate random values of r and Θ directly using procedures set forth in this paper. However, it should be noted that if sample point locations are selected using values of r and Θ randomly generated in this manner, not all possible sample point locations have the same probability of selection. Consequently, that method is not recommended.

This procedure works well for areas which are rectangular or square in shape. It is possible to implement the procedure in areas of other shapes, but randomly selected sample point locations falling outside the area must be rejected. Frequent replacement of rejected sample point locations can make the procedure somewhat inefficient.

If the area being sampled is large or has an irregular boundary, considerable time might be saved through selecting random coordinates in advance and ordering them in a sequence, thus minimizing the total distance traveled. For large sample sizes, a computer-generated numerical solution would be best (Ashley and Beers, 1972).

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