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Site Index Conversion Equations for Western Redcedar and Western Hemlock

Abstract

Equations that predict site index for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) from the site index of western redcedar (*Thuja plicata* Donn ex D. Don) (or vice versa) are needed to make good forest management decisions in the Pacific North West. Site index data for hemlock and redcedar were collected from sample plots located in two major ecological subzones in southern coastal British Columbia. Geometric mean regression was used to estimate the parameters of a linear model relating the site indices of hemlock and redcedar. The correlation between the site indices was high, indicating a good fit to the linear model. In general, the site index of hemlock is higher than the site index of redcedar, indicating that hemlock is more productive than redcedar on a given site. A validation procedure showed that one set of conversion equations was adequate for both ecological subzones. The conversion equations can be applied in mixed species stands or in stand conversion situations, although caution is required for both applications.

Introduction

Western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) and western redcedar (*Thuja plicata* Donn ex D. Don) commonly grow in mixtures on the coast of northwestern North America. There are many similarities between these species (Burns and Honkala 1990). Their ranges extend from California to Alaska on the coast, and both species also grow in interior British Columbia, Washington, Idaho, and Montana. Redcedar and hemlock are very shade tolerant and both are considered to be late succession species. As their ranges are quite similar, their sites have similar climatic and edaphic characteristics, although redcedar can grow on drier sites than hemlock. When grown in mixed stands, hemlock will usually overtop redcedar.

Western redcedar and western hemlock are often found in mixed stands, making growth and yield information on redcedar and hemlock mixtures important. Site index, which in British Columbia is top height at breast height age 50, is a particularly important piece of information because it is a measure of site quality. Site index is used along with stand density to estimate volume yields. Therefore, site index estimates for both species are required to estimate their yields when they both occupy the same site.

Site index may not be estimable for both species when redcedar and hemlock are growing in a mixed stand. This would be the case when height and age data are missing for one species, or if the trees of one species have some early suppression or are not in the dominant or upper co-dominant layer. Situations may also arise where a stand is harvested and a different species lacking site index data is regenerated on the site. In these cases, site index conversion equations may be used to estimate the site index of one species from the site index of another species (Carmean 1972, Hägglund 1981, Nigh 1995).

This paper describes the development of site index conversion equations for western redcedar and western hemlock on the southern coast of British Columbia.

Study Area

The study area was comprised of two subzones in the Coastal Western Hemlock (CWH) biogeoclimatic zone in British Columbia (Meidinger and Pojar 1991). The study plots were located north and east of Vancouver, B.C., in two variations of the CWH zone: the dry maritime (dm) subzone and in the submontane very wet maritime subzone (vm1) (Green and Klinka 1994).

The dm subzone has relatively dry summers, whereas the vm1 subzone has relatively wet cool summers. The dm subzone has a similar climate to the *Pseudotsuga menziesii*-*Tsuga heterophylla* forest, and the vm1 subzone to the *Picea sitchensis*-*Tsuga heterophylla* forest as delineated by Franklin and Dyrness (1973) for Washington and Oregon.

Methods

Study Plots

Sixty-three 0.04 ha plots were established in stands with both western hemlock and western redcedar trees. Nineteen of the 63 plots were located in the dm subzone and 44 plots were in the vm1 subzone. The samples were established in stands that had been logged and burned in the early 1900s. The emerging natural regeneration after burning contained extensive areas of even-aged western redcedar stands with minor components of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco var. *menziesii*) and western hemlock.

Within each study plot, five dominant trees of each species were selected as the sample trees. The height and breast height age of the sample trees were measured and recorded. The heights and ages were averaged and the height-age models for western redcedar and western hemlock (Nussbaum 1996) were used to estimate site index. This resulted in site index pairs for the 63 plots.

Data Analyses

In a preliminary analysis, the site indices of western hemlock were plotted against the site indices of western redcedar. This graphical analysis suggested that a linear model relating the site indices of the two species was appropriate, where:

$$SI_{Hw} = m \times SI_{Cw} + b + \epsilon, \text{ or} \quad (1)$$

$$SI_{Cw} = m \times SI_{Hw} + b + \epsilon. \quad (2)$$

SI_{Hw} is the site index at breast height age 50 for western hemlock, SI_{Cw} is the site index at breast height age 50 for western redcedar, m is the slope parameter, b is the intercept parameter, and ϵ is an error term.

The parameters, m and b , of the site index conversion equation were estimated using geometric mean regression (Leduc 1987, Nigh 1995, Ricker 1973, 1984). This method was chosen because we required a system of two invertible

linear regression models, that is, two models whereby one regression model can be produced from the other by mathematically inverting it.

The parameters of the geometric mean regression line, m and b , were estimated from the following equations (Leduc 1987, Nigh 1995), where:

$$\hat{m} = \text{sgn}(r) \times \frac{S_Y}{S_X}, \quad (3)$$

$$\hat{b} = \bar{Y} - \hat{m} \times \bar{X} \quad (4)$$

\hat{m} and \hat{b} are estimates of m and b , respectively, r is the correlation coefficient for the site index pairs, $\text{sgn}(r)$ is: +1 if r is positive, -1 if r is negative, and 0 if r is 0, S_Y and S_X are the standard deviations of the dependent and independent variables, respectively, and \bar{Y} and \bar{X} are the means of the dependent and independent variables, respectively. For model (1), $Y = SI_{Hw}$ and $X = SI_{Cw}$, and for model (2), $Y = SI_{Cw}$ and $X = SI_{Hw}$. Note that the parameters of models (1) and (2) can be estimated from the data or the parameters can be estimated for one model and derived by algebraic manipulation for the other model. Both methods give the same results.

Formulae for the 95% confidence intervals for the geometric mean regression parameters, \hat{m} and \hat{b} , are given in equations (5) and (6) (Leduc 1987 [but note the errors in equations [10] and [11] in this article], Rayner 1985), respectively,

$$\left[\hat{m} \times \sqrt{\frac{1-Q}{1+Q}}, \hat{m} \times \sqrt{\frac{1+Q}{1-Q}} \right], \text{ and} \quad (5)$$

$$\left[\hat{b} - t_{n-2,0.975} \times \sqrt{\frac{S_{YY}}{n} \times (1-r) \times \left(2 + \bar{X}^2 \times \frac{1+r}{S_{XX}} \right)}, \right. \\ \left. \hat{b} + t_{n-2,0.975} \times \sqrt{\frac{S_{YY}}{n} \times (1-r) \times \left(2 + \bar{X}^2 \times \frac{1+r}{S_{XX}} \right)} \right], \quad (6)$$

where $t_{n-2,0.975}$ is the 0.975th quantile of the Student's t distribution with $n-2$ degrees of freedom, n is the number of observations, S_{YY} and S_{XX} are the variances of the dependent and independent variables, respectively, and

$$Q = t_{n-2,0.975} \sqrt{\frac{1-r^2}{r^2 \times (n-2)}}. \quad (7)$$

Models (1) and (2) were fit to the whole data set (the combined models) and to the data for the dm and vm1 subzones individually (the dm and vm1 models). Cross-validation was done to

TABLE 1. Results of the geometric mean regression analysis.

Data set	Model	Parameter	Parameter			r
			Estimate	95% confidence interval		
				Lower bound	Upper bound	
Combined	$SI_{Hw} = m \times SI_{Cw} + b$	\hat{m}	1.048	0.8441	1.301	0.82
		\hat{b}	1.256	-3.502	6.014	
	$SI_{Cw} = m \times SI_{Hw} + b$	\hat{m}	0.9545	0.7688	1.185	
		\hat{b}	-1.199	-5.944	3.546	
dm	$SI_{Hw} = m \times SI_{Cw} + b$	\hat{m}	0.8437	0.6549	1.087	0.90
		\hat{b}	7.478	2.597	12.36	
	$SI_{Cw} = m \times SI_{Hw} + b$	\hat{m}	1.185	0.9198	1.527	
		\hat{b}	-8.864	-16.50	-1.229	
vm1	$SI_{Hw} = m \times SI_{Cw} + b$	\hat{m}	1.164	0.9065	1.495	0.79
		\hat{b}	-2.074	-7.790	3.642	
	$SI_{Cw} = m \times SI_{Hw} + b$	\hat{m}	0.8588	0.6688	1.103	
		\hat{b}	1.782	-2.795	6.369	

determine if the combined model was adequate or if we require a separate model for the dm and vm1 subzones. Each observation was sequentially removed from a dataset, models (1) and (2) were re-fit, and the error in the estimated cedar and hemlock site index was calculated for the removed observation. The errors were tested for normality with the W test (Shapiro and Wilk 1965) and for bias with a t-test. The F_{max} statistic (Hartley 1950, Neter et al. 1990) was used to test whether the dm and vm1 models resulted in a significantly different variance in the errors in estimated site index for the dm and vm1 data.

Results

Table 1 shows the results of the geometric mean regression for models (1) and (2) fit to the combined, dm, and vm1 data. The mathematical equivalence of the fitted models (1) and (2) can be easily demonstrated by inverting one model and noticing that the parameters are the same (with tolerance for rounding errors) as the parameters in the complementary model.

Figures 1 and 2 are plots of the data points overlaid on the combined and dm/vm1 models, respectively. These figures show a strong relationship between the site indices of western redcedar and western hemlock when they are growing in a mixed stand. This observation is verified by the high correlation coefficients (Table 1).

The results of the model testing are given in Table 2. The W statistic indicates that the errors

in estimated site index are normally distributed. This result is important because the t-test for bias and the F_{max} test both assume that the data are normally distributed. The t-tests show that all models are unbiased, that is, the mean errors from the models are not statistically significantly different from zero. The critical value for the F_{max} test at a significance level of $\alpha = 0.05$ is approximately 2.62 and 1.90 for two populations with 18 and 43 degrees of freedom, respectively. For

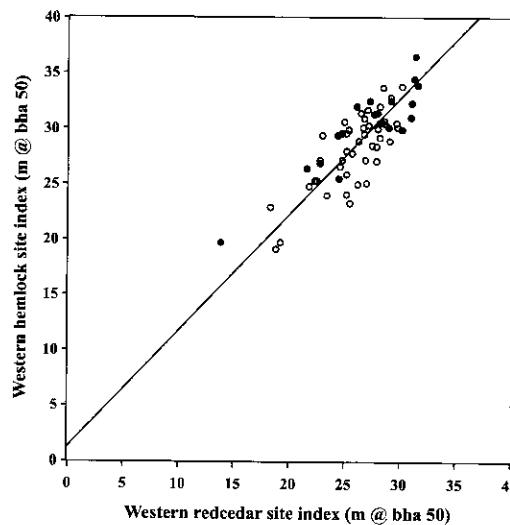


Figure 1. Western hemlock and western redcedar site indices from the dm subzone (•) and vm1 subzone (◦) with the fitted geometric mean regression line (—) estimated from all the data.

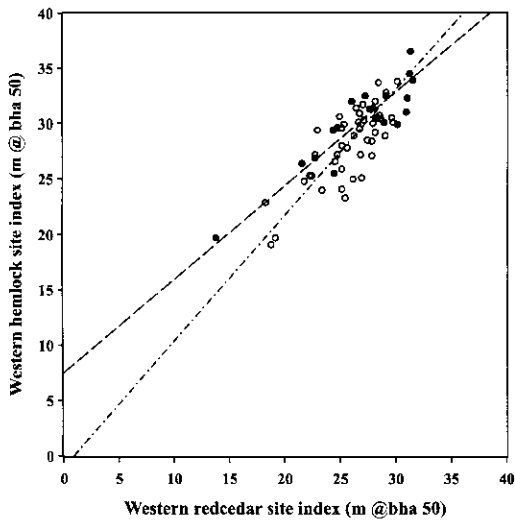


Figure 2. Western hemlock and western redcedar site indices from the dm subzone (•) and vm1 subzone (◊) with the fitted geometric mean regression line estimated from the dm subzone data (---) and from the vm1 subzone data (-.-).

the cedar data, $F_{\max} = 1.10$ (4.84+4.39) and 1.11 (4.27+3.83) for the dm and vm1 subzones, respectively. For the hemlock data, $F_{\max} = 1.46$ (4.98+3.40) and 1.12 (5.21+4.66) for the dm and vm1 subzones, respectively. These results indicate that the variances of the errors from the combined and the dm/vm1 models are not significantly different. We conclude from the t-test and the F_{\max} tests that the combined model is not biased, nor does it have poorer precision when compared to the dm and vm1 models, and therefore is adequate for both subzones.

Discussion

This study clearly shows that the site index of western redcedar can be estimated from the site index of western hemlock, and vice versa, with the following models:

$$SI_{Hw} = 1.048 \times SI_{Cw} + 1.256, \text{ and} \quad (8)$$

$$SI_{Cw} = 0.9545 \times SI_{Hw} - 1.199. \quad (9)$$

Site index conversion equations (8) and (9), developed with data from both the dm and vm1 subzones, are adequate for both subzones. The equations are applied by obtaining the site index for either western hemlock or western redcedar. The known site index is then input into either equation (8) or (9), depending on whether the site index of redcedar or hemlock is known, to estimate the site index of the complementary species. The conversion equations are designed specifically for mixed species stands. However, they can also be used for stand conversion situations, that is, when a stand of one species is harvested and regenerated with another species. In this situation, species interactions will not be present so the estimated site index may be biased.

The use of the conversion equations is contingent on the availability of an unbiased site index estimate for one of the species in the stand. This site index can be obtained from trees whose height growth has not been impeded by non-site factors, such as suppression or disease, or from some other method such as soil-site relationships. If a biased site index is used in the model, then the converted site index will be biased as well. Even when a good site index estimate is available, the conversion equations must be applied judiciously. For example, the species for which a site index is being

TABLE 2. Results of the model testing. The dm and vm1 datasets have 19 and 44 observations, respectively. The p-values for the t-test and W statistic are in brackets.

Model	Dataset	Western redcedar error				Western hemlock error			
		Mean	Variance	t-test	W	Mean	Variance	t-test	W
Combined	dm	-0.719	4.39	-1.50 (0.152)	0.959 (0.553)	0.763	4.98	1.49 (0.153)	0.963 (0.629)
	vm1	0.303	4.27	0.971 (0.337)	0.977 (0.651)	-0.309	4.66	-0.951 (0.347)	0.977 (0.650)
dm	dm	-0.0547	4.84	-0.108 (0.915)	0.949 (0.382)	0.0374	3.40	0.0884 (0.931)	0.944 (0.319)
vm1	vm1	-0.00295	3.83	-0.0100 (0.992)	0.981 (0.783)	0.0100	5.21	0.0290 (0.977)	0.980 (0.750)

estimated may not attain the potential height implied by the site index because of, for example, suppression. Practitioners should also be aware that there is an error associated with the site index estimate from any conversion equation. Therefore, the precision of the site index estimate is degraded when conversion equations are applied sequentially. This may be the case when a site index conversion from species A to C is desired but a conversion equation is not available. Therefore, a practitioner may have to convert the site index of species A to B, then from species B to C, but the cost will be reduced precision.

The site index of the western hemlock was consistently higher than the site index of the western redcedar. This is in agreement with previous observations that indicate that redcedar is often overtopped by hemlock when grown in mixed stands

(Burns and Honkala 1990). This result also demonstrates that site index is a species specific concept. Even though the two species are growing on the same site, they do not grow at the same rate and hence do not have the same productivity on a given site.

Conclusion

This study makes site index conversion equations accessible for western hemlock and western redcedar. These conversion equations will be important in good forest management because hemlock and redcedar are common species on the coast of northwestern North America. The data for the equations were collected from two major ecological subzones, and one conversion equation is adequate for both subzones.

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